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Common Ownership among Private Firms and Privatization Policies*

Junichi Haraguchi[†] and Toshihiro Matsumura[‡]

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Abstract

This study investigates the relationship between the optimal privatization policy and the degree of common ownership among private firms by formulating a mixed oligopoly model in which one public firm competes against private firms under common ownership. We find that depending on the private firms' cost structure, one of the following three patterns emerges: (a) the optimal degree of privatization is increasing in the degree of common ownership, (b) the optimal degree of privatization is decreasing in the degree of common ownership, (c) an inverted U-shaped relationship exists between the two. If the marginal cost of private firms is constant, then (b) always emerges, regardless of whether the marginal cost of the public firm is increasing or constant. However, if the marginal cost of private firms is increasing, then all three patterns can emerge. Our results suggest that the property of the optimal privatization policy depends crucially on the cost structure of private firms.

JEL classification numbers: H44, L13, L32, K21

Key words: overlapping ownership, optimal degree of privatization, mixed oligopolies, relative profit maximization, payoff interdependence

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1 Introduction

Institutional investors such as Vanguard, BlackRock, and Fidelity own shares in many major listed firms (Azar et al., 2018), which makes them common owners of the major firms in the same industries. If firms are concerned about the interests of these common owners, then they are indirectly concerned about other firms' profits. This means that the firms' payoffs are dependent on each other. Hence, they may deviate from profit-maximizing behavior. Partial ownership by common owners in the same industries may internalize externalities and improve welfare. López and Vives (2019) point out that common ownership internalizes the spillover effect of R&D and may accelerate welfare-improving R&D. The concern with an upstream monopolist about a downstream monopolist may mitigate the double-marginalization problem in a vertically related industry, and thus, common ownership may improve welfare. Sato and Matsumura (2019c) show that common ownership affects entry incentives and may improve welfare by reducing the inefficiency caused by excessive entries. However, common ownership reduces firms' incentives to compete in product markets, which is often harmful to welfare (Azar et al., 2018). Moreover, such collusive behavior may affect optimal public policies such as industrial, trade, and privatization policies. Common ownership has become a central issue in recent debates on antitrust policies because the degree of common ownership has grown substantially over recent years (Elhauge, 2016). Some empirical studies show that common ownership substantially affects firms' strategic behavior.¹

Common ownership is also increasing in mixed oligopoly type industries in which state-owned public firms compete with firms owned by private sectors, such as banking, telecommunications, transportation, and energy. Despite the global trend of state-owned public enterprises moving toward privatization, public enterprises with substantial ownership shares from the public sector remain active in various industries and manage a significant share of the world's resources (Megginson and Netter, 2001; La Porta et al., 2002). According to an OECD report, over 10% of the 2,000 largest companies worldwide are public enterprises, with sales equivalent to approximately 6% of

¹See Backus et al. (2019) for an example of the rise in common ownership in the US and Schmalz (2018) for a review of empirical studies that suggest a link between common ownership and firms' behavior.

global GDP (Kowalski et al., 2013). Particularly, Japanese, French, Chinese, Russian, Vietnamese, and Indian public enterprises occupy significant positions in many industries (Gupta, 2005; Huang and Yang, 2016; Huang et al., 2017; Fridman, 2018; Liu et al., (2020); Matsumura and Haraguchi, 2020d). The privatization of these state-owned firms is an important global issue. Reflecting the importance of privatization policies, the literature on mixed oligopolies is rich and diverse.² Nevertheless, to the best of our knowledge, no study investigates how common ownership affects the optimal privatization policy.

In this study, we investigate how common ownership among private firms affects the optimal privatization policy. In other words, we investigate how industry competitiveness affects the welfare-maximizing privatization policy of the state firms in the same industry. We use linear-quadratic cost functions that cover two popular models in the literature on mixed oligopolies: formulations with constant marginal costs and formulations with quadratic costs. Depending on the cost parameters, we find three patterns: (a) The optimal degree of privatization is increasing in the degree of common ownership. (b) The optimal degree of privatization is decreasing in the degree of common ownership. (c) The relationship between the optimal degree of privatization and common ownership is inverted U-shaped.

Furthermore, (b) always emerges if the private firm's marginal cost is constant, regardless of whether the public firm's marginal cost is constant or increasing. Additionally, all three patterns can emerge if the private firm's marginal cost is increasing. Our results suggest that the relationship between the optimal privatization policies and ownership structure among private firms is complex. This relationship depends crucially on the cost structure of private firms.

Our study is related to that of Matsumura and Okamura (2015), who adopt the payoff interdependence approach (relative profit maximization) and investigate the relationship between the industry competitiveness and the optimal privatization policies.³ They show that the optimal degree of privatization is increasing (decreasing) in the competitiveness of the market under constant

²See Heywood and Ye (2009), Lee et al. (2018), and Futagami et al. (2019).

³For a discussion of the relative profit maximization approach, see Matsumura and Matsushima (2012) and Matsumura et al. (2013).

marginal costs (quadratic costs).

However, our study is different from theirs in two regards. First, they consider a scenario with tougher competition than in the case of profit-maximizing private firms, whereas we address a scenario with weaker competition than in the case of profit-maximizing private firms.⁴

Second, we consider a broader class of cost functions. We therefore find that the cost structure of private firms rather than that of public firms is crucial for the result, which they did not derive in their work.

Our study is also related to the literature on mixed oligopolies that discusses the relationship between the competitiveness of markets and privatization policies. De Fraja and Delbono (1989) show that privatization is more likely to improve welfare when the number of private firms is larger. Lin and Matsumura (2012) and Matsumura and Okamura (2015) demonstrate that the optimal degree of privatization is increasing in the number of private firms in various contexts.⁵ However, Haraguchi and Matsumura (2020c) show that the optimal degree of privatization can be increasing in the asymmetry among private firms. This result has the opposite implication as those from the three works above because an increase in the asymmetry among firms (the number of firms) increases (decreases) market concentration indexes such as the HHI. We show that the relationship between industry competitiveness and privatization policies is further complicated, and suggest the importance of investigating the cost structures of private firms.

The paper proceeds in four sections. Section 2 introduces the model. Section 3 presents an equilibrium analysis. Section 4 reports the results. Finally, Section 5 concludes the paper.

2 The model

We consider a mixed oligopoly in which one public firm (firm 0) owned initially by the government competes with n private firms (firms 1, 2,..., n). We focus on the effect of common ownership

⁴Kim. et al. (2019) discuss the relationship between the optimal degree of privatization and corporate social responsibility among private firms. Like Matsumura and Okamura (2015), they study the case in which competition among private firms is tougher than that in profit-maximizing private firms, in contrast to the case of common ownership.

⁵For the competition and privatization policy, see also Anderson et al. (2000).

among private firms to privatization policy. We thus suppose that $n \geq 2$. Let q_i be firm i 's output. The firms produce homogeneous products for which the inverse demand function is $p(Q) = a - Q$, where p denotes price, a is a positive constant, and $Q := \sum_{i=0}^n q_i$. We assume that all private firms have an identical cost function, although we allow the public and private firms to have different cost functions.⁶ We assume linear-quadratic cost functions. The public firm's cost function is $c_0(q_0) = \gamma_0 q_0 + (\kappa_0/2)q_0^2$ and each private firm i 's cost function is $c_i(q_i) = \gamma q_i + (\kappa/2)q_i^2$, where $\gamma_0, \kappa_0, \gamma$, and κ are non-negative constants. We assume that $\gamma_0 \geq \gamma$. This model formulation of demand and cost covers several popular settings in the literature on mixed oligopolies. For example, in De Fraja and Delbono's (1989) model, $\gamma_0 = \gamma = 0$ and $\kappa_0 = \kappa > 0$.⁷ Additionally, in Pal's (1998) model, $\gamma_0 > \gamma$ and $\kappa_0 = \kappa = 0$.⁸ Our model covers these two popular models in the literature on mixed oligopolies. As Matsumura and Okamura (2015) show, these two models can yield opposite policy implications in mixed oligopolies, and we thus believe that a model formulation covering these two models is important.

Following the standard assumption in this field, we define social welfare as the sum of the consumer surplus and profits of firms. Social welfare W is

$$W = \int_0^Q p(q) dq - pQ + \sum_{i=0}^n \pi_i = \int_0^Q p(q) dq - \sum_{i=0}^n c_i(q_i).$$

Following Matsumura (1998), we assume that the public firm's objective function is $v_0 = \alpha \pi_0 + (1 - \alpha)W$.⁹ $\alpha \in [0, 1]$ represents the degree of privatization. Following the recent theoretical literature on common ownership (e.g., López and Vives, 2019), we assume that each private firm i has the following objective function is $v_i = \pi_i + \lambda(\Pi^p - \pi_i)$, where Π^p is the sum of the private firms' profits (i.e., $\Pi^p - \pi_i$ is the sum of the private rivals' profits) and $\lambda \in [0, 1]$ represents the degree of common ownership.¹⁰

⁶For the endogenous cost difference between public and private firms, see Matsumura and Matsushima (2004).

⁷De Fraja and Delbono (1989) assume that the public and private firms have identical cost functions; however, the models allowing cost differences between the public and private firms are also widely used (Matsumura and Shimizu, 2010; Kawasaki et al., 2020).

⁸See also Mujumdar and Pal (1998) and Haraguchi and Matsumura (2020a,b).

⁹For empirical evidence supporting this formulation, see Ogura (2018). Additionally, see Seim and Waldfogel (2013) for empirical evidence for $\alpha = 0$.

¹⁰If we allow a negative λ , we can study the case in which competition is tougher than in the case of profit-

The game runs as follows. In the first stage, the government chooses α to maximize social welfare. In the second stage, each firm simultaneously chooses its output to maximize its objective function. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

Throughout this study, we assume interior solutions in the second stage. In other words, all firms produce positive outputs.

3 Equilibrium

First, we solve the second stage game given α . The first order-condition for each private firm is

$$p + p'q_i - \gamma - \kappa q_i + p'\lambda \left(\sum_{i=1}^n q_i - q_i \right). \quad (1)$$

The second-order condition is satisfied. The first-order condition of the public firm is

$$p + \alpha p'q_0 - \gamma_0 - \kappa_0 q_0. \quad (2)$$

The second-order condition is satisfied.

These first-order conditions yield the following equilibrium quantities for the public and private firms in the second stage subgames:

$$q_0^S(\alpha, n) = \frac{(a - \gamma_0)(1 + (n - 1)\lambda + \kappa) - n(\gamma_0 - \gamma)}{(1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0))}, \quad (3)$$

$$q^S(\alpha, n) = \frac{(a - \gamma)(\alpha + \kappa_0) + \gamma_0 - \gamma}{(1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0))}. \quad (4)$$

Superscript S indicates the equilibrium outcomes in the second-stage subgame. We obtain the

maximizing private firms and discuss the relationship between the industry competitiveness and optimal privatization policies more generally. For the rationale for the negative λ , see Matsumura and Matsushima (2012) and Matsumura et al. (2013). All our results hold when λ is negative and not too small.

following equilibrium total output, price, each private firms' profit, and welfare:

$$Q^S(\alpha, n) = \frac{n(a - \gamma)(\alpha + \kappa_0) + (1 + (n - 1)\lambda + \kappa)(a - \gamma_0)}{(1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0))}, \quad (5)$$

$$p^S(\alpha, n) = \frac{(1 + (n - 1)\lambda + \kappa)(a(\alpha + \kappa_0) + \gamma_0) + n\gamma(\alpha + \kappa_0)}{(1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0))}, \quad (6)$$

$$\pi^S(\alpha, n) = \frac{2(1 + (n - 1)\lambda) + \kappa}{2} \left(\frac{(\alpha + \kappa_0)(a - \gamma) + \gamma_0 - \gamma}{(1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0))} \right)^2, \quad (7)$$

$$W^S(\alpha, n) = \frac{X_1}{2((1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0)))^2}. \quad (8)$$

We report X_1 and the other coefficients (X_i) that appear throughout the study in Appendix A.

We now present some properties of the equilibrium outcomes in the second stage subgame (i.e., the properties of the equilibrium outcome given α).

Lemma 1 (i) p^S is increasing in λ . (ii) π^S is increasing in λ if and only if

$$(n - 1)(1 - \lambda)(\alpha + \kappa_0) - (1 + (n - 1)\lambda) > 0. \quad (9)$$

(iii) W^S is decreasing in λ .

Proof See Appendix B.

Lemma 1(i) states that common ownership restricts competition among private firms and raises the price, which harms consumers. Lemma 1(ii) states that common ownership increases the private firms' profits under the plausible condition, but not always. Because the left-hand side in (9) is decreasing in λ , an increase in λ is more likely to increase the profits of private firms when λ is smaller, and an increase in λ always reduces the profits when λ is close to one. An increase in λ raises the price and increases firm 0's output. The latter effect reduces the profit of private firms and may dominate the former effect. Lemma 1(iii) states that an increase of common ownership always reduces welfare given α .

Next, we discuss the government's welfare maximization problem in the first stage. Let α^F be the equilibrium degree of privatization (superscript F indicates the first stage). The first-order condition is

$$\frac{\partial W^S(\alpha)}{\partial \alpha} = -\frac{((1 + (n - 1)\lambda + \kappa)(a - \gamma_0) - n(\gamma_0 - \gamma))X_2}{((1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0)))^3} = 0. \quad (10)$$

The second-order condition

$$-\frac{((1 + (n - 1)\lambda + \kappa)^2 + n\kappa)^4}{((1 + (n - 1)\lambda + \kappa)(a - \gamma_0) - n(\gamma_0 - \gamma))^2(X_3)^3} < 0$$

is satisfied under the assumption of an interior solution in the second stage (i.e., both q_0 and q are positive). The solution to (10), α^* , is

$$\alpha^* = \frac{n(1 + (n - 1)\lambda)(\gamma_0 - \gamma + \kappa_0(a - \gamma))}{((1 + (n - 1)\lambda + \kappa)^2 + n\kappa)(a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)}. \quad (11)$$

The equilibrium α , α^F , is

$$\alpha^F = \max\{0, \min\{\alpha^*, 1\}\}.$$

In other words, if the solution is interior (i.e., $\alpha^F \in (0, 1)$), then $\alpha^F = \alpha^*$.

4 Results

The analysis above leads to the following result.

Proposition 1 (i) $\alpha^F > 0$ (i.e., the optimal degree of privatization is strictly positive). (ii) If $\kappa_0 = \kappa > 0$, $\gamma_0 = \gamma$, and $(1 + (n - 1)\lambda + \kappa)^2 - \lambda\kappa n(n - 1) > 0$, then $\alpha^F < 1$ (i.e., if both the public and private firms have the same cost function, then the optimal degree of privatization is less than one unless both λ and n are sufficiently large). (iii) α^* is increasing in γ_0 . (iv) α^* is decreasing in γ . (v) α^* is increasing in κ_0 . (vi) α^* is decreasing in κ . (vii) α^* is increasing in n (the optimal degree of privatization is increasing in the number of private firms). (viii) α^* is increasing in λ if and only if

$$g(\lambda) := (1 + (n - 1)\lambda)^2(a - \gamma_0) - (n + \kappa)(\kappa(a - \gamma_0) - n(\gamma_0 - \gamma)) < 0. \quad (12)$$

Proof See Appendix B.

Proposition 1(i) has already been demonstrated under the general demand and cost conditions when $\lambda = 0$ (Matsumura, 1998). From Proposition 1(i), the solution is interior if $\alpha^F < 1$. Matsumura (1998) shows that if the public and private firms have the same cost function, then the

optimal degree of privatization is less than one when $\lambda = 0$. Proposition 1(ii) states that this is not true when both λ and n are sufficiently large.¹¹

Proposition 1(iii)-(vi) suggests that the optimal degree of privatization is more likely to be higher when the public firm has a more significant cost disadvantage over private firms, which is a natural and intuitive result. Remember that an increase in the degree of privatization induces production substitution from the public firm to the private firms, and this production substitution is more likely to improve welfare when the private firms are more efficient than the public firm.¹²

Proposition 1(vii) states that the optimal degree of privatization is increasing in the number of private firms. An increase in n increases the total output, which reduces the total output effect. An increase in n strengthens the production substitution effect. Both effects increase the optimal degree of privatization.¹³

To understand the implications and intuitions behind Proposition 1(viii), which is our main result, we present a result on the properties of g .

Lemma 2 *(i) g is increasing in λ . (ii) g is independent of κ_0 . (iii) g is decreasing in κ . (iv) g is decreasing in γ . (v) g is increasing in γ_0 .*

Proof See Appendix B.

We now present a result highlighting the implications of Proposition 1(viii).

Proposition 2 *Suppose that the solution is interior (i.e., $\alpha^F < 1$). Then, (i) α^F is either (a) increasing in λ for any $\lambda \in [0, 1]$, (b) decreasing in λ for any $\lambda \in [0, 1]$, or (c) inverted U-shaped; (ii) (a) is more likely to hold when κ is larger, γ is larger, and γ_0 is smaller; (iii) (a) holds if $\gamma_0 = \gamma$ and $\kappa \geq \bar{\kappa} := (\sqrt{5} - 1)n/2$, (c) holds if $\gamma_0 = \gamma$ and $\kappa \in (\underline{\kappa}, \bar{\kappa})$ where $\underline{\kappa} := (\sqrt{n^2 + 4} - n)/2$, and (b) holds if $\gamma_0 = \gamma$ and $\kappa \leq \underline{\kappa}$; (iv) (b) always holds if $\kappa = 0$.*

Proof See Appendix B.

We now explain the intuition behind our results. An increase in α directly reduces the public

¹¹The optimal degree of privatization can also be zero in a free entry market (Matsumura and Kanda, 2005), in the presence of the excess cost of public funds (Sato and Matsumura, 2019b), or if the government chooses a privatization policy over time (Sato and Matsumura, 2019a).

¹²For a general discussion of welfare-improving production substitution, see Lahiri and Ono (1988).

¹³We explain the production substitution effect and the total output effect in detail after Proposition 2.

firm's output, which indirectly increases the private firms' outputs through strategic interaction. Because the public firm is concerned about consumer surplus, the public firm's price–cost margin is less than that for each private firm. That is, the public firm's marginal cost is greater than that of each private firm. Thus, the production substitution from the public firm to the private firms improves welfare (the production substitution effect). However, an increase in α reduces total output, which reduces consumer surplus (the total output effect). This trade-off determines the optimal degree of privatization, α^F .

Suppose that κ is positive (i.e., the private firms' marginal costs are increasing). An increase in λ reduces the output of each private firm, which reduces each firm's marginal cost. This strengthens the welfare-improving production substitution effect mentioned above. Therefore, an increase in λ may increase the optimal degree of privatization, especially when κ is larger. However, an increase in λ reduces total output, and thus increases welfare loss due to the suboptimal production level. Therefore, when λ reaches a threshold value, the total output effect dominates the production substitution effect, and thus, a further increase in λ reduces the optimal degree of privatization. This threshold value exceeds one (is smaller than zero, lies on $(0,1]$) when κ is large (small, moderate). Thus (a) ((b), (c)) emerges when κ is large (small, moderate).

When γ_0 is large and γ is small, the production substitution effect is strong, regardless of λ . Thus, the impact of λ on the production substitution effect is less important. Under these conditions, (b) is more likely to hold when γ_0 is larger and γ is smaller.

Proposition 2 indicates that common ownership reduces the optimal degree of privatization if the private firms' technology has constant returns to scale, whereas common ownership increases the optimal degree of privatization if the private firms' technology has significant decreasing returns to scale. If private firms' technology has insignificant decreasing returns to scale, then the degree of common ownership and the optimal privatization policy has a non-monotone relationship, and the optimal degree of privatization is the highest when the degree of common ownership is moderate. Our results suggest that we should pay more attention to the cost structure of private firms when we discuss privatization policies.

Finally, we present a result on the relationship between common ownership and welfare.

Proposition 3 *Suppose that the solution is interior (i.e., $\alpha^F < 1$). Then, W^F is decreasing in λ .*

Proof See Appendix B.

An increase in common ownership restricts competition among private firms and reduces welfare. Lemma 1(iii) states that this result holds when α is exogenous and Proposition 3 states that it holds even when α is endogenous.

5 Concluding remarks

In this study, we investigate mixed oligopolies in which one public firm competes against private firms under common ownership. For this purpose, we formulate models with common ownership among private firms and examine the relationship between the optimal privatization policy and the degree of common ownership among private firms. We find that three kinds of relationship may emerge between them: (a) the optimal degree of privatization is increasing in the degree of common ownership, (b) the optimal degree of privatization is decreasing in the degree of common ownership, and (c) these have an inverted U-shaped relationship. If the private firms have a constant marginal cost, then (b) always emerges, regardless of whether the marginal cost of the public firm is increasing or constant. If the private firms' marginal cost is increasing, however, then all three patterns can emerge. Our results suggest that the property of the optimal privatization policy depends crucially on the cost structure of the private firms. Thus, the government should pay more attention to the cost structures of private firms rather than that of the public firms when it chooses privatization policies. We also find that common ownership always reduces welfare and may even reduce private firms' profits. This result could suggest that investing in private firms in mixed oligopolies may be inefficient for institutional investors.

However, this study has some limitations. First, we neglect any externalities that yield another distortion. As López and Vives (2019) and Sato and Matsumura (2019c) show, common ownership may internalize externalities and mitigate welfare losses, and thus common ownership may improve

welfare. Incorporating externalities and extending our analysis is an avenue for future research.¹⁴

Second, we assume that all firms are owned by domestic investors.¹⁵ We presume that foreign ownership of private firms reduces the optimal degree of privatization, and we would obtain a similar relationship between the optimal degree of privatization and the degree of common ownership. Future studies may extend research in this regard.

¹⁴For possible sources of the divergence of social and private marginal costs in mixed oligopolies, see Haraguchi and Matsumura (2020d).

¹⁵For pioneering works on the nationality of private firms in mixed oligopolies, see Corneo and Jeanne (1994), Fjell and Pal (1996), and Pal and White (1998). Foreign ownership is important in the context of public policies in mixed oligopolies. See also Bárcena-Ruiz and Garzón (2005a, b).

Appendix A

$$\begin{aligned}
X_1 &:= ((1 + (n - 1)\lambda + \kappa)(a - \gamma_0) + n(a - \gamma)(\alpha + \kappa_0))^2 + (2\alpha + \kappa_0)((1 + (n - 1)\lambda + \kappa)(a - \gamma_0) \\
&\quad - n(\gamma_0 - \gamma))^2 + n(2(1 + (n - 1)\lambda) + \kappa)((\alpha + \kappa_0)(a - \gamma) + \gamma_0 - \gamma)^2, \\
X_2 &:= \alpha[((1 + (n - 1)\lambda + \kappa)^2 + n\kappa)(a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)] \\
&\quad - n(1 + (n - 1)\lambda)(\gamma_0 - \gamma + \kappa_0(a - \gamma)), \\
X_3 &= (1 + (n - 1)\lambda + \kappa)^2 + \kappa_0(\kappa + (1 + \lambda)n)^2 + \kappa_0(1 - \lambda)(1 - \lambda + 2(\kappa + n(1 + \lambda))) + n\kappa, \\
X_4 &= [(1 + \lambda - \kappa)^2 + n(2\lambda(1 - \lambda) + \kappa(1 + 2\lambda) + 2\kappa_0(1 - \lambda) + \kappa_0\kappa + n^2(\lambda^2 + (1 + 2\lambda)\kappa_0))](a - \gamma_0)^2 \\
&\quad + [n(2(1 - \lambda) + \kappa) + n^2](\gamma_0 - \gamma)^2 \\
X_5 &= (1 + \kappa_0)(1 - \lambda + \kappa)^2 + (2\lambda(1 - \lambda) + \kappa(1 + 2\lambda) + 2\kappa_0(1 + \lambda)(1 - \lambda + 2\kappa)n + (\kappa_0(1 + \lambda)^2 + \lambda^2)n^2).
\end{aligned}$$

Appendix B

Proof of Lemma 1

From (6), we obtain

$$\frac{\partial p^S}{\partial \lambda} = \frac{n(n-1)(\kappa_0 + \alpha)((\kappa_0 + \alpha)(a - \gamma) + \gamma_0 - \gamma)}{((1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0)))^2} > 0.$$

This implies Lemma 1(i).

From (7), we obtain

$$\frac{\partial \pi^S}{\partial \lambda} = \frac{(n-1)((\kappa_0 + \alpha)(a - \gamma) + \gamma_0 - \gamma)^2((n-1)(1 - \lambda)(\kappa_0 + \alpha) - (1 + (n-1)\lambda))}{((1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0)))^3}.$$

This is positive if and only if $(n-1)(1 - \lambda)(\kappa_0 + \alpha) - (1 + (n-1)\lambda) > 0$. This implies Lemma 1(ii).

From (8), we obtain

$$\frac{\partial W^S}{\partial \lambda} = -\frac{n(n-1)((\kappa_0 + \alpha)(a - \gamma) + \gamma_0 - \gamma)A}{((1 - \lambda + \kappa)(1 + \alpha + \kappa_0) + n(\lambda + (1 + \lambda)(\alpha + \kappa_0)))^3},$$

where $A := (1 + (n-1)\lambda)((\kappa_0 + (\alpha + \kappa_0)^2)(a - \gamma) + (1 + 2\alpha + \kappa_0)(\gamma_0 - \gamma)) - \alpha(\kappa_0(a - \gamma_0) - n(\gamma_0 - \gamma))$.

$(\partial W^S)/(\partial \lambda) < 0$ if and only if $A > 0$. $A > 0$ for any $\lambda \geq 0$. This implies Lemma 1(iii). ■

Proof of Proposition 1

From (10), we obtain

$$\frac{\partial W}{\partial \alpha}|_{\alpha=0} = \frac{(1 + (n-1)\lambda)((a - \gamma)\kappa_0 + \gamma_0 - \gamma)((a - \gamma_0)(1 + (n-1)\lambda + \kappa) - n(\gamma_0 - \gamma))}{((1 - \lambda + \kappa)(1 + \kappa_0) + n(\kappa_0 + \lambda(1 + \kappa_0)))^3}.$$

Because we suppose that $q_0^S > 0$, then $(a - \gamma_0)(1 + (n-1)\lambda + \kappa) - n(\gamma_0 - \gamma) > 0$. Therefore, $(\partial W)/(\partial \alpha)|_{\alpha=0} > 0$. This implies Proposition 1(i).

Suppose that $\kappa_0 = \kappa > 0$ and $\gamma_0 = \gamma$. From (10), we obtain

$$\frac{\partial W}{\partial \alpha}|_{\alpha=1} = -\frac{(a - \gamma)^2(1 + (n-1)\lambda + \kappa)((1 + (n-1)\lambda + \kappa)^2 - n(n-1)\lambda\kappa)}{((\kappa + 2)(1 + (n-1)\lambda + \kappa) + n(1 + \kappa))^3},$$

which is negative if $(1 + (n-1)\lambda + \kappa)^2 - n(n-1)\lambda\kappa > 0$. This implies Proposition 1(ii).

From (11), we obtain

$$\begin{aligned}
\frac{\partial \alpha^*}{\partial \gamma_0} &= \frac{n(a - \gamma_0)(1 + (n - 1)\lambda)((1 + \kappa_0)(1 + (n - 1)\lambda + \kappa)^2 + 2n\kappa_0(1 + (n - 1)\lambda + \kappa) + n\kappa + n^2\kappa_0)}{[(1 + (n - 1)\lambda + \kappa)^2 + n\kappa](a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)]^2} > 0, \\
\frac{\partial \alpha^*}{\partial \gamma} &= -\frac{n(a - \gamma_0)(1 + (n - 1)\lambda)((1 + \kappa_0)(1 + (n - 1)\lambda + \kappa)^2 + n(\kappa + 2\kappa_0(1 - \lambda + \kappa)) + n^2\kappa_0(1 + 2\lambda))}{[(1 + (n - 1)\lambda + \kappa)^2 + n\kappa](a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)]^2} < 0, \\
\frac{\partial \alpha^*}{\partial \kappa_0} &= \frac{n(a - \gamma)(1 + (n - 1)\lambda)}{((1 + (n - 1)\lambda + \kappa)^2 + n\kappa)(a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)} > 0, \\
\frac{\partial \alpha^*}{\partial \kappa} &= -\frac{n(\kappa_0(a - \gamma) + \gamma_0 - \gamma)(1 + (n - 1)\lambda)((2(1 + (n - 1)\lambda + \kappa)(a - \gamma_0) + n(a - 2\gamma_0 + \gamma))}{[(1 + (n - 1)\lambda + \kappa)^2 + n\kappa](a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)]^2} < 0.
\end{aligned}$$

These imply Proposition 1(iii–vi).

From (11), we obtain

$$\frac{\partial \alpha^*}{\partial n} = \frac{(\kappa_0(a - \gamma) + \gamma_0 - \gamma)B}{[(1 + (n - 1)\lambda + \kappa)^2 + n\kappa](a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)]^2},$$

where $B := [\lambda((1 - \lambda)\lambda + (1 + 2\lambda)\kappa)(a - \gamma_0) + (1 - \lambda(1 + \kappa))(\gamma_0 - \gamma)]n^2 + \lambda(1 - \lambda + \kappa)^2(a - \gamma_0)n + (1 + (n - 1)\lambda)(1 - \lambda + \kappa)^2(a - \gamma_0)$. $(\partial \alpha^*)/(\partial n) > 0$ if and only if $B > 0$. $B > 0$ for any $\lambda \geq 0$.

This implies Proposition 1(vii).

From (11), we obtain

$$\frac{\partial \alpha^*}{\partial \lambda} = -\frac{n(n - 1)(\kappa_0(a - \gamma) + \gamma_0 - \gamma)g(\lambda)}{[(1 + (n - 1)\lambda + \kappa)^2 + n\kappa](a - \gamma_0) - (n^2 + 2n(1 + (n - 1)\lambda) + n\kappa)(\gamma_0 - \gamma)]^2}. \quad (13)$$

This is positive if and only if $g(\lambda) < 0$. This implies Proposition 1(viii). ■

Proof of Lemma 2

From (12), we obtain

$$\frac{\partial g}{\partial \lambda} = 2(1 + (n - 1)\lambda)(a - \gamma) > 0, \quad (14)$$

$$\frac{\partial g}{\partial \kappa_0} = 0, \quad (15)$$

$$\frac{\partial g}{\partial \kappa} = -[(a - \gamma_0)(n + 2\kappa) - n(\gamma_0 - \gamma)] < 0, \quad (16)$$

$$\frac{\partial g}{\partial \gamma} = -n(n + \kappa) < 0, \quad (17)$$

$$\frac{\partial g}{\partial \gamma_0} = (n + \kappa + 1 + (n - 1)\lambda)(n + \kappa - (1 + (n - 1)\lambda)) > 0. \quad (18)$$

These imply Lemma 2. ■

Proof of Proposition 2

Lemma 2(iii–v) implies Proposition 2(ii).

Suppose that $\gamma_0 = \gamma$. Because $g(\lambda)$ is increasing in λ , $g(\lambda) < 0$ for any $\lambda \in [0, 1]$ if $g(1) = (n^2 - \kappa(n + \kappa))(a - \gamma) < 0$. Solving the equation $g(1) = 0$ with respect to κ and focusing on a positive solution, we obtain $\bar{\kappa} = (\sqrt{5} - 1)n/2$.

(c) holds if $g(0) < 0$ and $g(1) > 0$. $g(1) > 0$ holds if $\bar{\kappa} < 0$. Solving the equation $g(0) = (1 - (n + \kappa)\kappa)(a - \gamma) = 0$ with respect to κ and focusing a positive solution, we obtain $\underline{\kappa} = (\sqrt{n^2 + 4} - n)/2$.

Because $g(\lambda)$ is increasing in λ , $g(\lambda) > 0$ for any $\lambda \in [0, 1]$ if $g(0) > 0$. $g(0) > 0$ holds if $\kappa < \underline{\kappa}$. These imply Proposition 2(iii).

Substituting $\kappa = 0$ into $g(\lambda)$, we obtain $g(\lambda) = (1 + (n - 1)\lambda)^2(a - \gamma_0) + n^2(\gamma_0 - \gamma) > 0$. This implies Proposition 2(iv).

$g(\lambda)$ is increasing in λ for $\lambda \in [0, 1]$. Then, $g(\lambda)$ is negative for any $\lambda \in [0, 1]$ if $g(1) < 0$, which corresponds to case (a). $g(\lambda)$ is positive for any $\lambda \in [0, 1]$ if $g(0) > 0$, which corresponds to case (b). The equation $g(\lambda) = 0$ has a solution for $\lambda \in [0, 1]$ if $g(0) < 0$ and $g(1) > 0$, which corresponds to case (c). No other case exists. These imply Proposition 2(i). ■

Proof of Proposition 3

Suppose that $\alpha^F < 1$. Substituting α^* into (8) we obtain $W^F = X_4/(2X_5)$. We obtain

$$\frac{\partial W^F}{\partial \lambda} = -\frac{n(n-1)(\kappa_0(a-\gamma) + \gamma_0 - \gamma)^2(1 + (n-1)\lambda)(1 + (n-1)\lambda + n + \kappa)}{X_5^2} < 0.$$

This implies Proposition 3. ■

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